Transactional Justice

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1. Question

We intend to answer the following question: let us suppose that two agents A and B enter into a voluntary transaction. The transaction produces a certain surplus over the sum of the resources initially committed. How should that surplus be distributed? Or, more precisely, what would be the normative meaning of each possible way of distributing it?

2. Nomenclature

Let us imagine a situation where the following characteristics can be quantified:

- R: Joint production by agents A and B cooperating.
- IA and IB: Input respectively contributed by each agent.
- CA and CB: Coefficients denoting the technological capacity of each agent. It can grow with I+D+i.
- S: Surplus of the co-operation. Necessarily S = R-IA-IB. Let us suppose S > 0.
- PA and PB: Claims (pretensions) of each agent over the surplus S.
- UA and UB: Part of the respective claims undisputed by the other agent.
- NA and NB: Negotiating power of each agent.
- D: deficit of the superavit with regard to the claims D = CA+CB - S. Let us suppose D > 0.
- SA and SB: Output finally obtained by each agent. Necessarily: S = SA+SB.

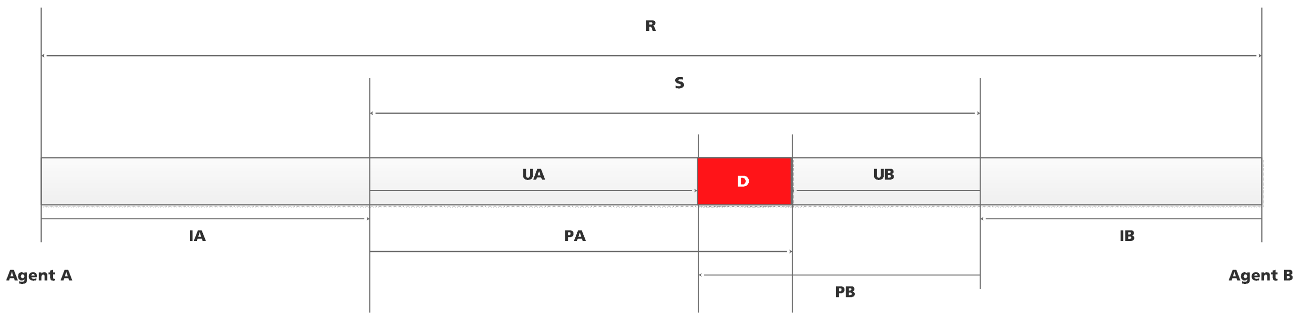

Figure 1

3. Production and distribution process

Let us suppose that agents A and B take part in the following process:

1. R is produced by A and B acting jointly, after investing IA and IB respectively. A surplus S is therefore resulting.
2. Each agent establishes a claim over the surplus: PA and PB. That claim reflects the agent’s concept of an adequate distribution. Several concepts are possible, depending on the moral quality of each agent and its interpretation of the situation. Therefore PA and PB need not be calculated on the same basis for both agents.
3. On the other hand, it may well happen well that $PA+PB > S$ (also any other possibility, but we shall focus initially on this). As a consequence, a substraction must happen, so that the agents will receive $SA < PA$ and/or $SB < PB$, for there is a deficit to be distributed over the sum of claims: $D = PA+PB - S$.

   There are also undisputed amounts: UA and UB. These are the parts of each claim that do not belong in the area under dispute.

4. The distribution of that deficit D is proportional to the negotiating power of each agent: $SA = UA + D * NA /(NA+NB)$ and $SB = UB + D * NB/(NA+NB)$. Obviously the sum is: $SA + SB = UA + UB + D = S$, and the arithmetic condition is fulfilled.

In summary, the amount to be distributed between the agents A and B is D:

- That amount is defined by the claims of the agents over the surplus, which embody their respective concepts of justice.
- Over the distribution of D a negotiation takes place where each’s one power determines the final amount of D (and in consequence of S) that will be appropriated by it. For our purpose, the negotiation power of each agent is the amount of resources it has at the beginning of the simulation tic (its "wealth", so to speak).

If $D = CA+CB-S < 0$, we have a trivial case where the distribution of D may follow the same rules.

4. Cardinality

Our model is essentially cardinal: values for agent A are added and substracted with values for agent B and with shared values (for example, the joint production). That implies that we are not talking about subjective utilities but something measurable in the same unit and comparable through both agents.

A (subjective) utility function would convert common values to individual utilities. It would then permit to formalize economic motivations. Leaving it aside has an explicative cost. However, part of that cost is covered by the fact that some part of the subjective inclinations of the agent (its concept of a just distribution) is included in the calculation of its claim over S.

It also contributes to make our use of a cardinal concept more palatable, the fact that we are moving at the highest possible level of generality. All amounts, inputs, production, negotiating powers, etc., are valued in generic resources which can be understood (or exchanged) as any possible economic good or service.

5. Three moral profiles

This essay intends to study the formation of those claims from a normative point of view.

5.1. Neoclassical

Doing so challenges the fundamental supposition of standard Neoclassical microeconomics: that all agents are similar, all of them maximizers of the resources under their possession.

- In our scheme, Neoclassical agents would uphold: PA = S and PB = S. As a consequence D = S, and the distribution of S between the agents would depend solely on their respective negotiation power.

The ultimatum game has shown repeatedly that in fact this is not the case. Ordinary people act with certain concepts of justice in the distribution of the aggregated value of a transaction, albeit different from person to person and from culture to culture. Even from a strategic point of view, acting as a pure Neoclassical agent is often counterproductive.

The standard Neoclassical approach includes however a certain concept of justice: the respect to the property rights of the other agent, that guarantee their voluntary enrolment in the transaction.

5.2. Criminal

A criminal approach would promise that respect to entice the other, but would sustain instead a claim PA > S (being A the criminal agent). If agent B has a low or null negotiation power, it may lose all resources committed to the transaction (IB).

This can be graphed:

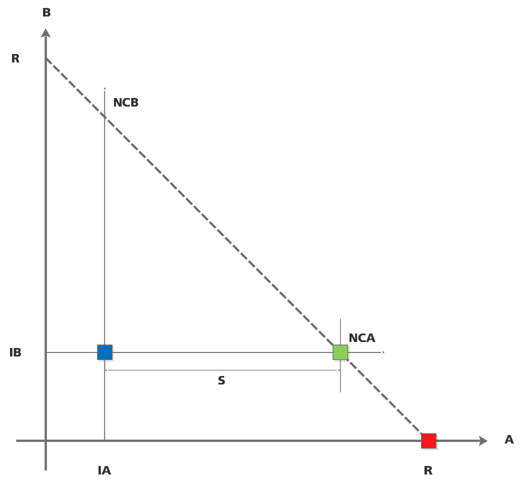

Figure 2

The blue square represents the initial position, before production, where agent A has committed IA and agent B, IB, to the operation. The dotted line represents the total amount produced in the transaction, that has to be shared somehow between A and B.

The green square represents the claim of agent A if it is a Neoclassical agent: all the surplus produced by the operation is then appropriated by A, while B appropriates the bare minimum to participate in the transaction (anything, never mind how small, over 0).

The red square represents the claim of agent A if it is a criminal agent: not only the surplus but also the initial amount committed by B (IB) is appropriated by A, which aspires then to get R in full.

5.3. Fair

Any pretension of agent A smaller than NCA or of agent B smaller than NCB, leads to a distribution of R that embodies a concept of justice different from the Neoclassical. The corresponding agent is the acting in a “benevolent” way, meaning that its claim is not the maximum that a Neoclassical agent would maintain.

But it is not a “benevolent dictator”. As we have said, there may be two such points, one corresponding to PA and another to PB. If as a result a deficit is created, which will be distributed according to the respective powers.

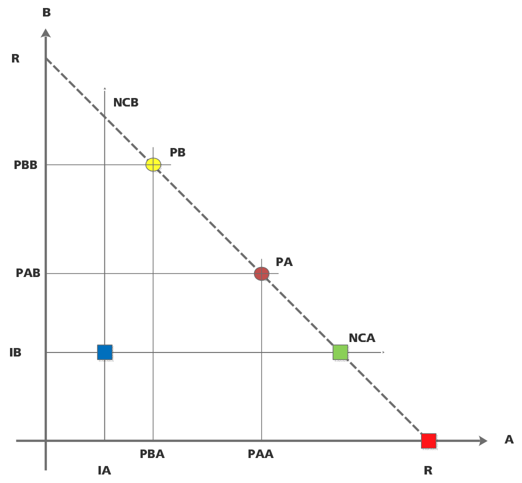

Figure 3

6. Production function

Let us suppose that the joint production of our agents happens through a CES production function. The assumtion of constant elasticity of substitution makes sense, for the sake of symmetry between the two agents.

A CES production function has the following form:

$$R = C * (\alpha * IA^{\mu} + (1 - \alpha) * IB^{\mu})^{\frac{d}{\mu}}$$

Where the meanings of R, IA and IB are the same already explained above (see 2), and:

- C is the efficiency due to technology, organization, etc.
- α expresses the influence of each factor over the production. It must be 0 < α < 1.
- μ expresses the elasticity of substitution between the factors: $\sigma = \frac{1}{1-\mu}$
- d is the degree of the production function (economies of scale implies d > 1).

For our simulation, it makes sense to set:

- C = CA+CB, (or C = CA*CB), where CA and CB express the respective levels of technology (see 2)
- $\alpha = \frac{CA}{CA+CB}$

Then we would have the following production function:

$$R(IA, IB) = (CA + CB) * (\frac{CA}{CA + CB} * IA^{\mu} + \frac{CB}{CA + CB} * IB^{\mu})^{\frac{d}{\mu}}$$

or, if we use the product of CA by CB:

$$R(IA, IB) = (CA * CB) * (\frac{CA}{CA + CB} * IA^{\mu} + \frac{CB}{CA + CB} * IB^{\mu})^{\frac{d}{\mu}}$$

7. Fair distribution

A fair distribution of the total production R between agents A and B may follow the general idea of proportionality, a concept of fairness suitable for pure market relations. Each agent receives in the proportion it has contributed to create the joint product.

(There are of course other possible concepts of justice, based on needs, on equality, on social rank, etc., but all of them happen to be external to the operation of creation of value implied in a market transaction. Proportionality according to contribution is not).

A good way to estimate the proportional parts is the following:

$$R = RA + RB$$

$$\frac{RA}{RB} = \frac{\frac{\partial R}{\partial IA}}{\frac{\partial R}{\partial IB}}$$

this is: each agent receives a part proportional to the contribution to the production of R of an infinitesimal increase in the amount of resources committed, divided by the same contribution of the other agent.

From there it is easy to obtain the fair pretension of the fair agent:

$$PA = RA - IA$$

The value of $\frac{RA}{RB}$ happens to be identical in the two versions of a CES production function we have indicated above:

$$\frac{RA}{RB} = \frac{CA * IA^{\mu-1} * IB^{1-\mu}}{CB}$$

That proportion does not depend on d.

In our simulation, IA = IB = 1, which means that a fair agent would aspire to a distribution:

$$\frac{RA}{RB} = \frac{CA}{CB}$$

independent also on the value of μ.